3. CALCULATIONS

The results of the calculations based on the method and program described above are contained in tables 1 through 12 and graphs 1 through 24. The odd-numbered graphs contain plots of the amplitude of F and F^* versus the distance from the transmitter on the bearing indicated, while the even-numbered graphs contain plots of the phase of F and F^* versus distance. Recall that F and F^* are the attenuation functions in the homogeneous and inhomogeneous cases, respectively.

To obtain the electric field from these results, one need only combine (12) and (15) with (3). The results are

$$\vec{E}(d) = i(I_0 \ell \mu_0 \omega/2\pi)\left(\exp(-i k_0 |\vec{d}|)/|\vec{d}|\right) F(d, Z)\hat{z} , \tag{23}$$

$$\vec{E}^*(d) = i(I_0 \ell \mu_0 \omega/2\pi)\left(\exp(-i k_0 |\vec{d}|)/|\vec{d}|\right) F^*(d, Z, Z_1)\hat{z} , \tag{24}$$

where $\vec{E}(d)$ is the electric field at a distance d over a homogeneous plane earth of impedance Z, while $\vec{E}^*(d)$ is that over an inhomogeneous earth. We can therefore write (23) and (24) approximately as

$$|E(d)| = \left(|E_0|/d\right) |F(d, Z)| , \tag{25}$$

$$\mathrm{Arg}\left(E(d)\right) = \mathrm{Arg}(E_0) + (\pi/2) - k_0 d + \mathrm{Arg}\left(F(d, Z)\right) , \tag{26}$$

$$|E^*(d)| = \left(|E_0|/d\right) |F^*(d, Z, Z_1)| , \tag{27}$$

$$\mathrm{Arg}\left(E^*(d)\right) = \mathrm{Arg}(E_0) + (\pi/2) - k_0 d + \mathrm{Arg}\left(F^*(d, Z, Z_1)\right) , \tag{28}$$

where E and E^* are the z-components of $\vec{E}$ and $\vec{E}^*$, respectively, Arg denotes the phase of a quantity, and E_0 can be taken to be approximately the electric (radiation) field (times $d \cong \lambda$, of course) generated by the actual antenna in its proximity ($d \cong \lambda$) on the desired bearing.

Returning to the results contained in section 7, we note that they represent the calculations for two distinct paths on bearings of 150° and 160° from a fixed transmitting site. In all cases the transmitter is taken to be at the U.S. Naval Reservation (38° 39' 17" N, 76° 31' 40" W) just north of Locust Grove Beach, Maryland, on the shore of the Chesapeake Bay. In what follows, reference to the map in figure 1 is suggested, and the parameter 'd' is the distance from the transmitter.

Graphs 1 through 8 and tables 1 through 4 are for a bearing of 160° E of N from the transmitter. The four cases contained in this set are for frequencies of 10, 15, 20, and 25 MHz, in that order. On this path, Chesapeake Bay is assumed to be homogeneous with electrical parameters of

$$\sigma = 2.0 \text{ mho/m}, \quad \epsilon = 81.0\ \epsilon_o ,$$

while the perturbing inhomogeneity is a section of land 6.85 km long ($28.3 \text{ km} \le d \le 35.15 \text{ km}$; 38° 24' 55" N, 76° 25' 00" W to 38° 21' 28" N, 76° 23' 24" W) across the Cove Point, Maryland area, which is assumed to have the electrical parameters of

$$\sigma_1 = 0.002 \text{ mho/m}, \quad \epsilon_1 = 15.0\ \epsilon_o .$$

This path ends in the Church Neck, Virginia area ($d \cong 142.57$ km; 37° 26' 51" N, 75° 58' 31" W).

Graphs 9 through 16 and tables 5 through 8 are for a bearing of 150° from the transmitter. Again the four cases are for frequencies of 10, 15, 20, and 25 MHz. Chesapeake Bay has the same parameters as above, while the perturbing inhomogeieity is a section of an island

4.32 km long (84.18 km $\leq$ d $\leq$ 88.5 km; 37° 59' 51' N, 76° 02' 50'' W to 37° 57' 50'' N, 76° 01' 23'' W) across the Smith Island, Maryland area which is assumed to have the electrical parameters of (marsh)

$$\sigma_1 = 1.0 \text{ mho/m}, \quad \epsilon_1 = 48.0\ \epsilon_o .$$

This path ends in the Matcholank Creek, Virginia area (d $\cong$ 121.3 km; 37° 42' 32'' N, 75° 50' 17'' W). Because of the possibility of poor earth on this island in addition to marsh, the above calculations were repeated with values of the electrical parameters of

$$\sigma_1 = 0.002 \text{ mho/m}, \quad \epsilon_1 = 15.0\ \epsilon_o .$$

The results are contained in graphs 17 through 24 and tables 9 through 12.

Let us now note some of the rather prominent features of the results. In each case F and F* naturally agree up to the "island," but in crossing the "island," which is a poorer conductor and dielectric than the surrounding medium, the amplitude falls off very sharply and a large change of phase occurs. After having crossed the "island," the phase and amplitude begin to recover and seem to approach the homogeneous values asymptotically. Note (see, for example, graph 3) that in the cases where the amplitude change is most drastic, the amplitude rises rapidly in the region after the "island" and "peaks up" before beginning to decrease again and asymptotically approach the homogeneous values. This is the so-called "recovery" or "focusing" effect. A similar phenomenon is exhibited in the phase (see, for example, graph 4).

That the asymptotic approach to the homogeneous values is at least qualitatively what should be expected can be seen by an analogy. One can visualize this system as a transmission line of impedance Z_a with a section of line of impedance Z_b inserted somewhere in its length.

If both $|Z_b|$ and $ARG(Z_b)$ are larger than $|Z_a|$ and $ARG(Z_a)$, respectively, as is the case here, then one can easily see that the amplitude should drop and an added phase change should occur in crossing the Z_b section. Asymptotic recovery should occur as the Z_b section becomes a very small perturbation on the system. This latter condition is met when its length is much smaller than the distance between the Z_b section and the point of observation. Since ground-wave propagation can be considered in terms of a transmission line in that it transports energy from one point to another, the above considerations tend to confirm the qualitative features of the results.

The "recovery" or "focusing" effect seen in the amplitude of F^* in the region immediately following the "island" was first discovered and experimentally verified by Millington (1949a; 1949b) for propagation across a coastline. A theoretical explanation of the recovery of the amplitude was also given by Millington (1949c), though the question of the phase change was left open. Both the amplitude and phase recoveries have been treated theoretically by a number of investigators; some discussion of these effects can be found in a paper by Wait (1964). The phase recovery was finally confirmed by Pressey et al. in 1956. Elson (1949) remarks that the recovery phenomenon owes its existence to a vertical redistribution of energy near the boundaries between media, and that this redistribution is inevitable because the field must vary with height differently on either side of the boundary because of the differing electrical parameters. The height-gain function will therefore have a different form depending upon whether the ground is primarily a conductor or primarily a dielectric. For the frequency range considered in this report, sea water has a fairly constant nature as a quasi-conductor, while land changes from a poor conductor at the lower frequencies to a poor dielectric at the higher ones. A very rough measure of the redistribution due to these effects can be seen as follows. According to Wait (1964, p. 199), for low heights and sufficiently far from the coast line, the height-gain function has the approximate form

$$h(z) \cong 1 + ikz(Z/\eta_0) , \tag{29}$$

where Z is the surface impedance, $\eta_0 = 120\pi\,\Omega$, $k = 2\pi/\lambda$, and z is the height above the surface. Letting

$$Z = \mathrm{Re}(Z) + i\,\mathrm{Im}(Z) , \tag{30}$$

we obtain

$$h(z) \cong \left(1 - kz\,\mathrm{Im}(Z)/\eta_0\right) + ikz\,\mathrm{Re}(Z)/\eta_0 . \tag{31}$$

For $|z| << 1$, we then obtain the following approximate forms for the magnitude and phase of the height-gain function:

$$|h(z)| \cong 1 - \alpha z , \tag{32}$$

$$\mathrm{Arg}\left(h(z)\right) \cong \beta z , \tag{33}$$

where

$$\alpha = k\,\mathrm{Im}(Z)/\eta_0 , \quad \beta = k\,\mathrm{Re}(Z)/\eta_0 . \tag{34}$$

For Z_1 one obtains, similarly, h_1 depending upon α_1 and β_1. From the headings of the tabular results, λ, Z, and Z_1 can be obtained for each case. The results can be found in the table below.

Frequency	All paths		Paths 1 and 3		Path 2	
	α	β	α_1	β_1	α_1	β_1
10 MHz	2.4	2.5	6.3	53	3.4	3.5
15 MHz	4.5	4.6	6.4	80	6.3	6.5
20 MHz	6.8	7.1	6.4	108	9.6	10.0
25 MHz	9.5	10.0	6.5	134	13.0	14.0

A comparison of these magnitudes with the graphical results shows that the relative magnitude of the recovery of the phase and amplitude follows roughly the above pattern. This lends some credence to the redistribution explanation of the recovery effect. A more detailed analysis and proof is beyond the scope of this report, however.

Next, we wish to consider the effects of moving the transmitting antenna from an assumed site over water, across a coastline, to land. This part of the study (Rosich, 1969) was prompted because of the "island inhomogeneity" restriction of the model used. This restriction coupled with the desire to investigate the attenuation beyond the "island" forced the assumption that transmitting antenna was over water in the Chesapeake Bay. The reasons for this should be clear from the fore going discussion of the model. The actual situation, however, was that the transmitting antenna was behind the coastline on land. If we now are willing to give up values of the attenuation beyond what was previously our "island" on path 1, then we can again use the model to investigate the case where the transmitting antenna is behind the coastline on land. In this latter case, our "island" of inhomogeneity becomes the water between the two sections of land: that at the transmitter and that at the Cove Point, Maryland area. The results for 10 MHz for this path are shown in graphs 25 and 26. In these graphs, the horizontal axis is the distance of the receiver from the coastline, not from the transmitter as in the earlier graphs, and Δ is the distance of the transmitting antenna behind the coastline. Therefore, Δ plus the value on the horizontal axis is equal to the distance from the transmitter to the receiver. Graphs 27 through 30 present the percent changes in the amplitude and phase (relative to the values at Δ=0) at a given distance D (from the coastline) before and across the "island" of our previous discussion (the Cove Point, Maryland area). One can easily see from these graphs that the magnitudes of the changes are largest before the "island," while the percentage change is significant in both regions. In graphs 29 and 30 note particularly the peaks in the curves which are suggestive of some type of focusing effect. This focusing effect points up the fact that for each receiver location, D, there is an optional location, Δ, for the transmitting antenna. As noted above, unfortunately nothing can be said about the region following the "island" (of our previous discussion), as the model only allows a three-section path, where the first and last sections have the same electrical properties. This is a shortcoming in the model for the present case, but the results presented above should outline the importance of the actual antenna sites for high-frequency ground-wave propagation and permit some estimate of the quantitative nature of the effect.

Lastly, in order to prevent misunderstanding, a few questions should be anticipated. They are: (1) How valid are the results for $|\Delta|$ less than a few wavelengths, since the model ignores the static and induction fields of the antennas? (2) Is the case labeled "Δ=0" really this case, since the model ignores the land behind the coastline in this case? In answer to (1), the results of Wait (1963) show that the results should not be altered significantly except in a region $|\Delta| << 1$ (actually a skin depth or so) around the coastline. The only change even then is the removal of a singularity (already removed from the graphs) in the field at the boundaries of the media. Thus, for small values of $|\Delta|$ the results presented here are approximately correct or at least indicative of the behavior. Because of this, and the fact that the model ignores reflections from boundaries and the effect of any media "behind" the transmitter or receiver, the case labeled "Δ=0" should really be labeled "Δ slightly greater than a skin depth in front of the coastline." Since the skin depth here is between

1×10^{-5} and 1×10^{-6} m, "$\Delta=0$" should convey the correct meaning, however. Also, Wait (1963) showed that the reflection effects are relatively small, thus the results are good approximations. This answers objection (2).

4. RECOMMENDATIONS AND CONCLUSIONS

Since many of the recommendations concern the approximations made in the course of obtaining the solutions presented here, we shall first list these approximations, then discuss their advantages, validity, and limitations; finally we shall put forth a number of recommendations for possible improvements to this model. In making these approximations, we (1) assumed that the earth is essentially flat over the distances involved; (2) assumed that the antennas are short vertical electric dipoles; (3) ignored the static and induction fields of the antennas; (4) used the surface impedance concept; (5) assumed that there is only one "island" of the inhomogeneity imbedded in an otherwise homogeneous medium and that within each of these two regions the electrical parameters are constants.

For distances less than 100 km, the flat earth is probably a reasonable approximation, although the matter should be decided by a comparison of the attenuation function for a homogeneous earth in the flat and spherical cases. This would give a good indication of the range of validity of the approximation for the inhomogeneous case also, as it is computed as a perturbation of the homogeneous values.

The antennas in this model have been assumed, for simplicity, to be short vertical electric dipoles, while the actual antenna, as evidenced by its several interacting elements and its radiation pattern, will, in addition, have several higher order multipole terms. If